Perceptron

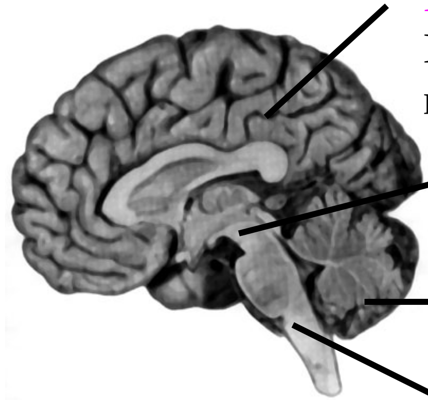
The human brain

Seat of consciousness and cognition

Perhaps the most complex information processing machine in nature

Historically, considered as a monolithic information processing machine

Beginner's Brain Map



Forebrain (Cerebral Cortex): Language, maths, sensation, movement, cognition, emotion

Midbrain: Information Routing; involuntary controls

Cerebellum: Motor Control

Hindbrain: Control of breathing, heartbeat, blood circulation

Spinal cord: Reflexes, information highways between body & brain

Brain : a computational machine?

Information processing: brains vs computers

- brains better at perception / cognition
- slower at numerical calculations
- parallel and distributed Processing
- Brain astonishing in the amount of information it processes
  - Typical computers: 10^9 operations/sec
  - Housefly brain: 10^{11} operations/sec

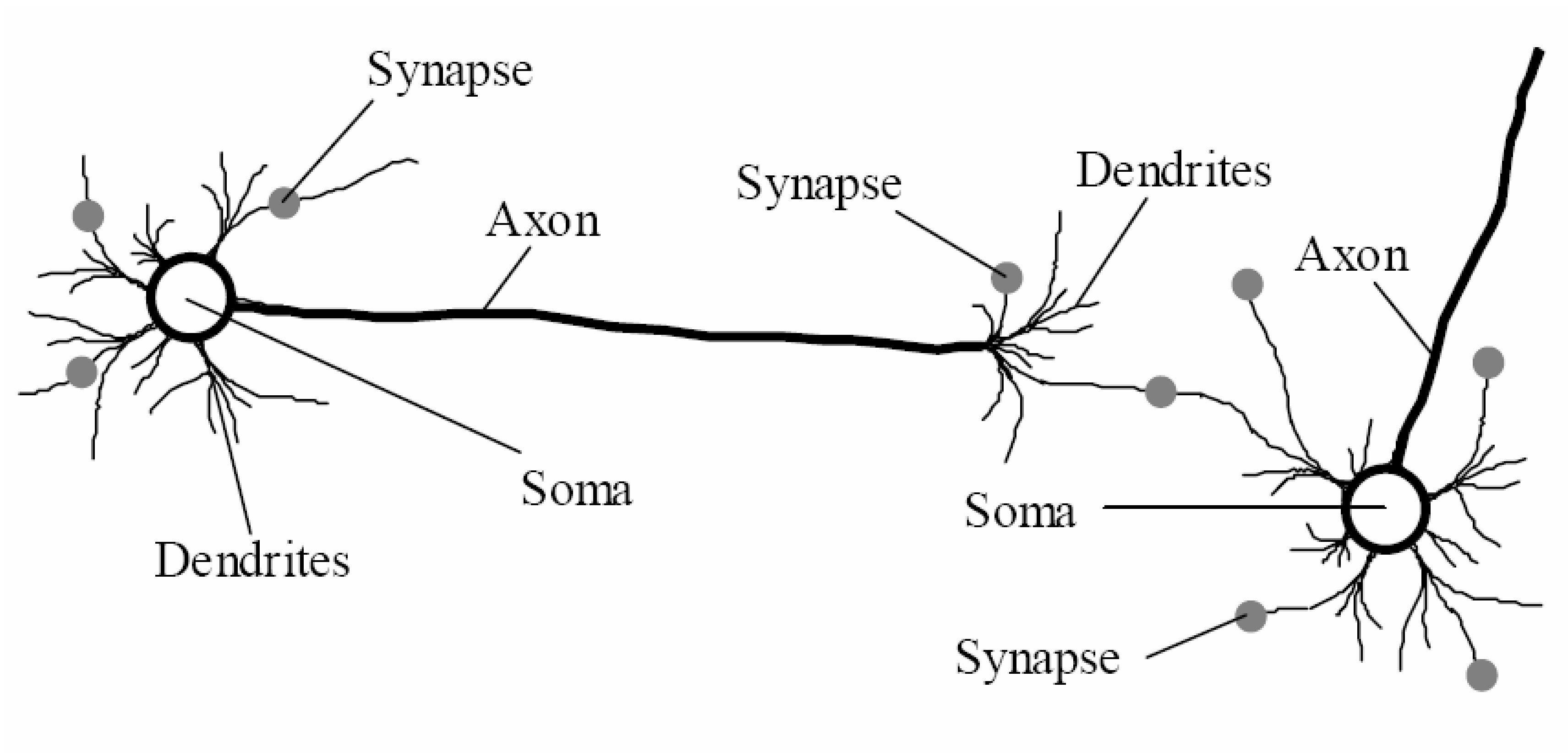
Brain facts & figures

- Basic building block of nervous system: nerve cell (neuron)
- ~ 10^{12} neurons in brain
- ~ 10^{15} connections between them
- Connections made at “synapses”
- The speed: events on millisecond scale in neurons, nanosecond scale in silicon chips

- A **neural network** can be defined as a model of reasoning based on the human brain. The brain consists of a densely interconnected set of nerve cells, or basic information-processing units, called **neurons**.
- The human brain incorporates nearly 10 billion neurons and 60 trillion connections, *synapses*, between them. By using multiple neurons simultaneously, the brain can perform its functions much faster than the fastest computers in existence today.

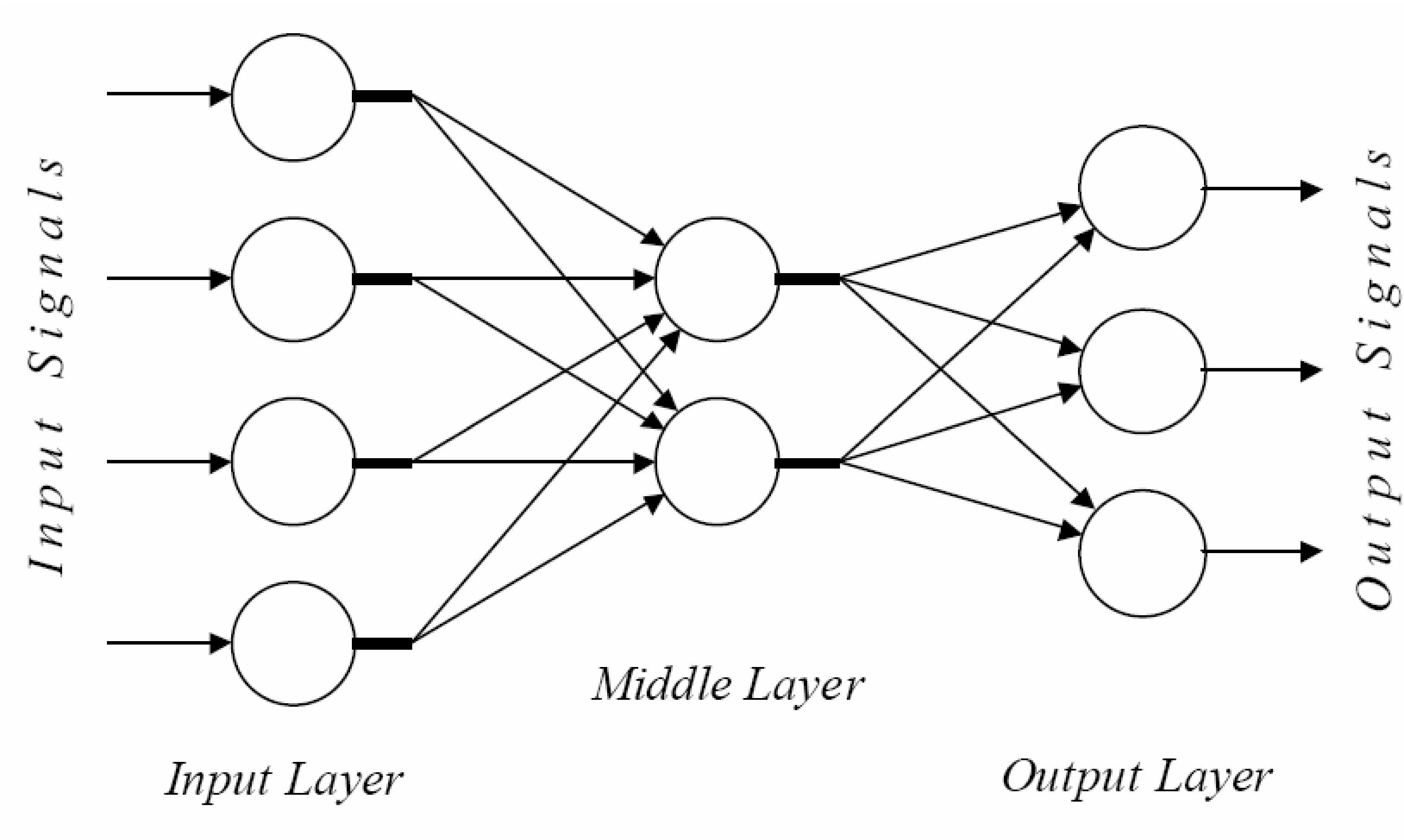
- Each neuron has a very simple structure, but an army of such elements constitutes a tremendous processing power.
- A neuron consists of a cell body, **soma**, a number of fibers called **dendrites**, and a single long fiber called the **axon**.

Biological neural network



- An artificial neural network consists of a number of very simple processors, also called **neurons**, which are analogous to the biological neurons in the brain.
- The neurons are connected by weighted links passing signals from one neuron to another.
- The output signal is transmitted through the neuron's outgoing connection. The outgoing connection splits into a number of branches that transmit the same signal. The outgoing branches terminate at the incoming connections of other neurons in the network.

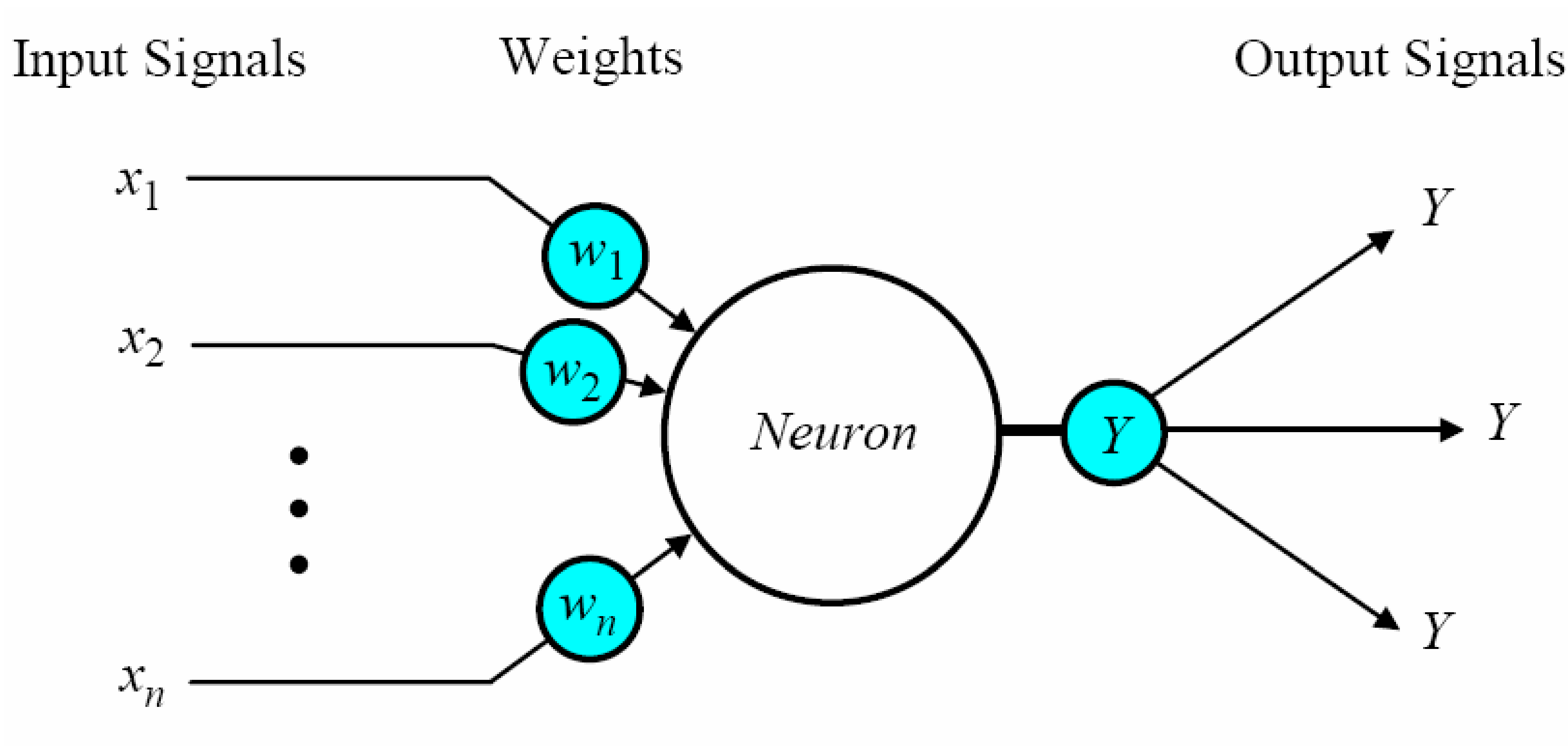
Architecture of a typical artificial neural network



Analogy between biological and artificial neural networks

Biological Neural Network	*Artificial Neural Network*
Soma	Neuron
Dendrite	Input
Axon	Output
Synapse	Weight

The neuron as a simple computing element



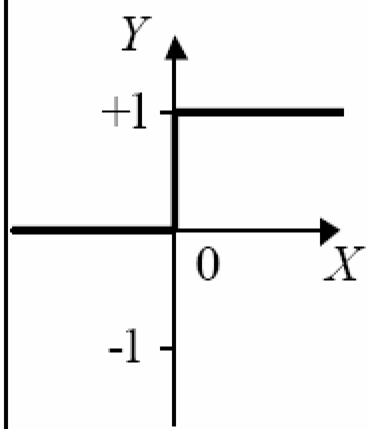
Activation Function

- The neuron uses the following transfer or The neuron uses the following transfer or **activation function**

$$X = \sum_{i=1}^{n} x_i w_i \qquad\qquad Y = \begin{cases} +1, & \text{if } X \geq \theta \\ -1, & \text{if } X < \theta \end{cases}$$

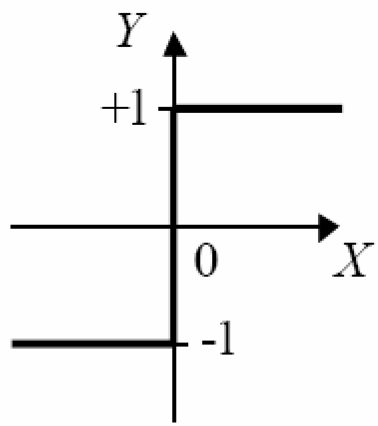
Activation Functions

Step function



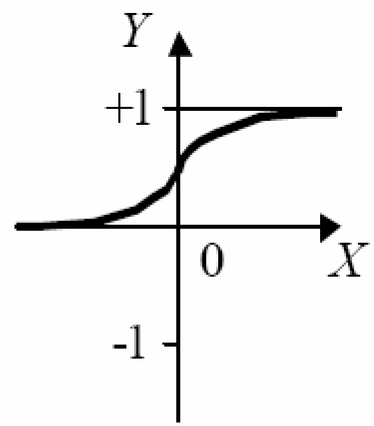
$$Y^{step} = \begin{cases} 1, & \text{if } X \geq 0 \\ 0, & \text{if } X < 0 \end{cases}$$

Sign function



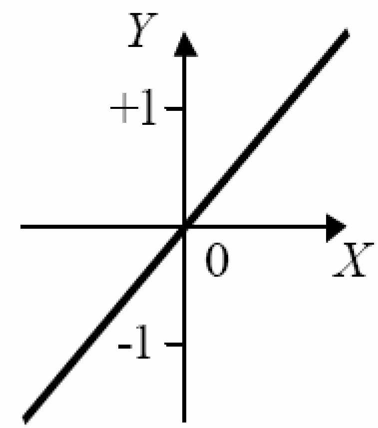
$$Y^{sign} = \begin{cases} +1, & \text{if } X \geq 0 \\ -1, & \text{if } X < 0 \end{cases}$$

Sigmoid function



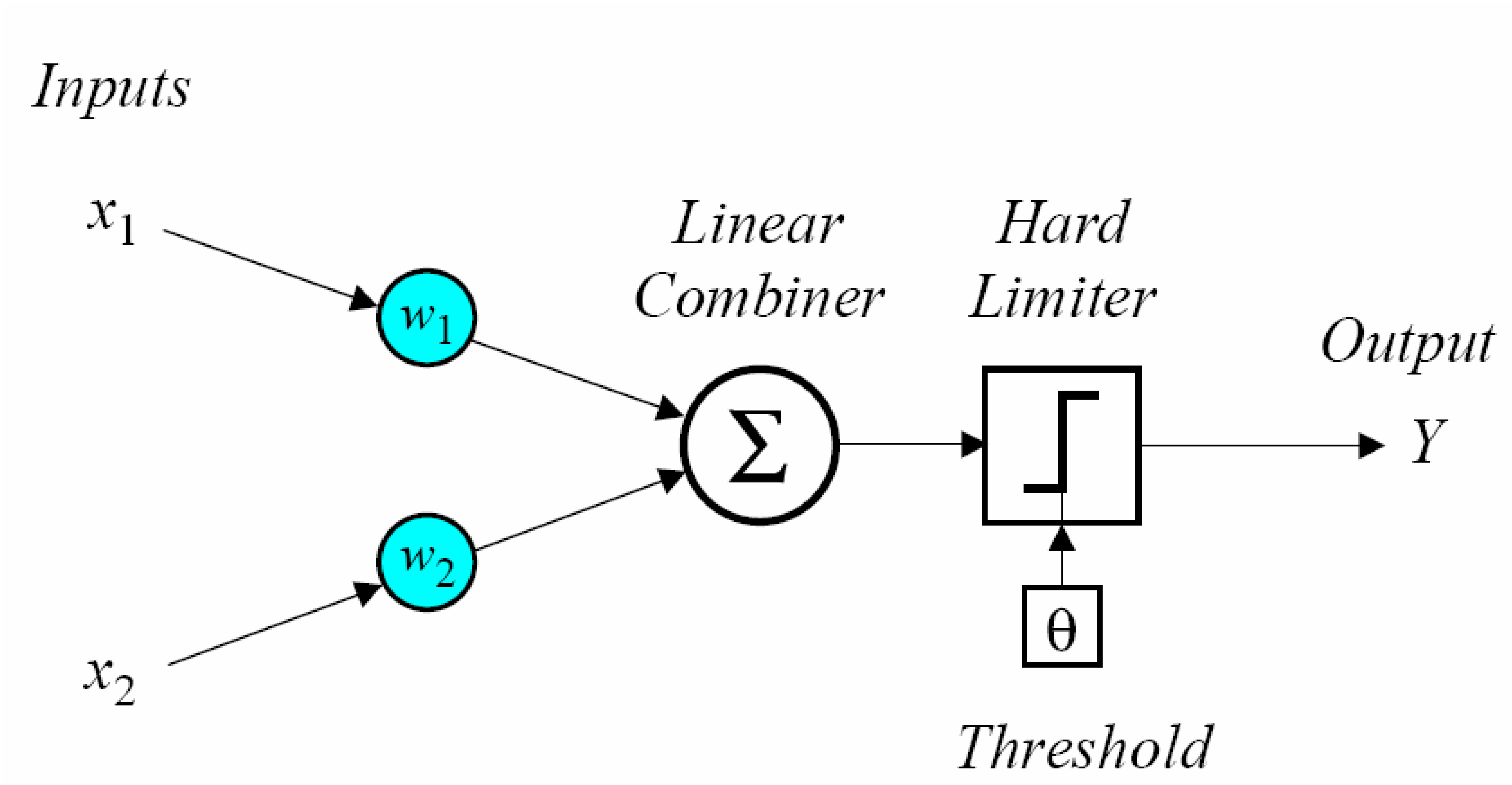
$$Y^{sigmoid} = \frac{1}{1+e^{-X}}$$

Linear function

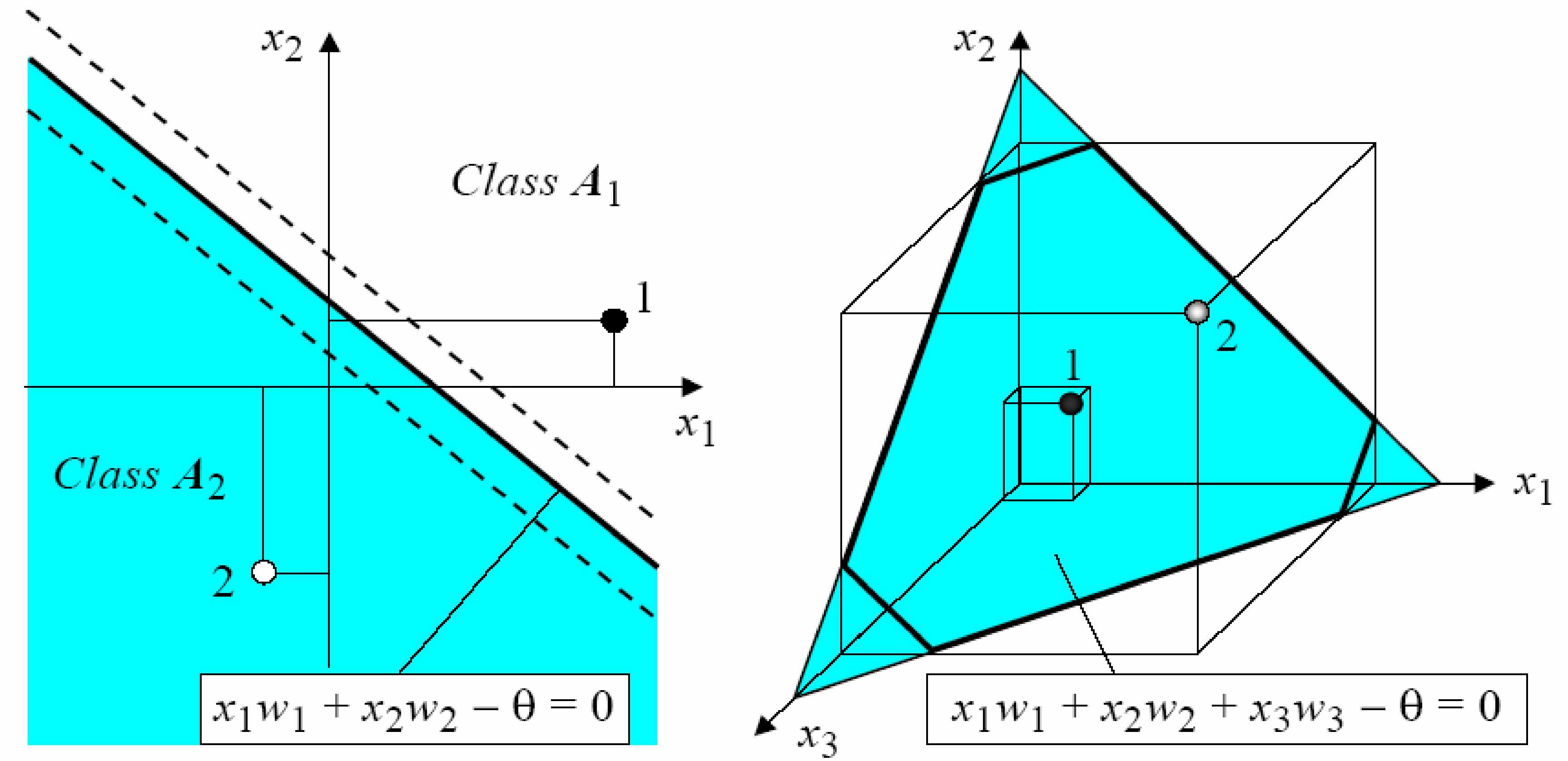


$$Y^{linear} = X$$

Single-layer two-input Single-layer two-input perceptron



Linear separability in the in the perceptrons



(a) Two-input perceptron. (b) Three-input perceptron.

- If at iteration p, the actual output is $Y(p)$ and the desired output is $Y_d(p)$, then the error is given by:

$$e(p) = Y_d(p) - Y(p)$$ where $p = 1, 2, 3, \ldots$

  Iteration p here refers to the pth training example presented to the perceptron.

- If the error, $e(p)$, is positive, we need to increase perceptron output $Y(p)$, but if it is negative, we need to decrease $Y(p)$.

The perceptron learning rule

$$w_i(p+1) = w_i(p) + \alpha \cdot x_i(p) \cdot e(p)$$

where $p = 1, 2, 3, \ldots$

α is the **learning rate**, a positive constant less than unity.

The perceptron learning rule was first proposed by **Rosenblatt** in 1960. Using this rule we can derive the perceptron training algorithm for classification tasks.

Perceptron's tarining algorithm

<u>Step 1</u>: Initialisation

Set initial weights $w_1, w_2, \ldots, w_n$ and threshold θ to random numbers in the range $[-0.5, 0.5]$.

If the error, $e(p)$, is positive, we need to increase perceptron output $Y(p)$, but if it is negative, we need to decrease $Y(p)$.

Perceptron's tarining algorithm (continued)

Step 2: Activation

Activate the perceptron by applying inputs $x_1(p)$, $x_2(p),\ldots, x_n(p)$ and desired output $Y_d(p)$. Calculate the actual output at iteration $p = 1$

$$Y(p) = step\left[\sum_{i=1}^{n} x_i(p)\, w_i(p) - \theta\right]$$

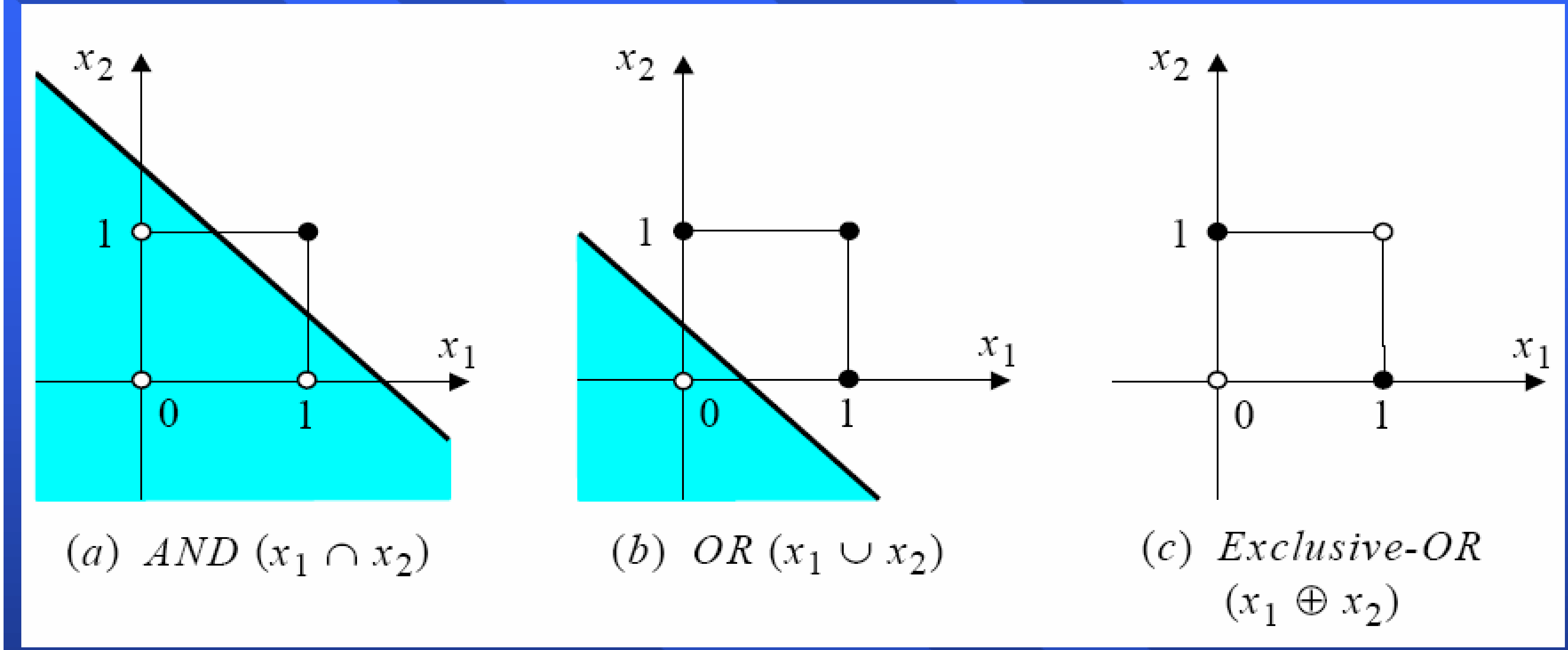
where n is the number of the perceptron inputs, and *step* is a step activation function.

Example of perceptron learning: the logical operation *AND*

Epoch	Inputs		Desired output	Initial weights		Actual output	Error	Final weights	
	x_1	x_2	Y_d	w_1	w_2	Y	e	w_1	w_2
1	0	0	0	0.3	–0.1	0	0	0.3	–0.1
	0	1	0	0.3	–0.1	0	0	0.3	–0.1
	1	0	0	0.3	–0.1	1	–1	0.2	–0.1
	1	1	1	0.2	–0.1	0	1	0.3	0.0
2	0	0	0	0.3	0.0	0	0	0.3	0.0
	0	1	0	0.3	0.0	0	0	0.3	0.0
	1	0	0	0.3	0.0	1	–1	0.2	0.0
	1	1	1	0.2	0.0	1	0	0.2	0.0
3	0	0	0	0.2	0.0	0	0	0.2	0.0
	0	1	0	0.2	0.0	0	0	0.2	0.0
	1	0	0	0.2	0.0	1	–1	0.1	0.0
	1	1	1	0.1	0.0	0	1	0.2	0.1
4	0	0	0	0.2	0.1	0	0	0.2	0.1
	0	1	0	0.2	0.1	0	0	0.2	0.1
	1	0	0	0.2	0.1	1	–1	0.1	0.1
	1	1	1	0.1	0.1	1	0	0.1	0.1
5	0	0	0	0.1	0.1	0	0	0.1	0.1
	0	1	0	0.1	0.1	0	0	0.1	0.1
	1	0	0	0.1	0.1	0	0	0.1	0.1
	1	1	1	0.1	0.1	1	0	0.1	0.1

Threshold: $\theta = 0.2$; learning rate: $\alpha = 0.1$

Two-dimensional plots of basic logical operations



A perceptron can learn the operations *AND* and *OR*, but not *Exclusive-OR*.